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EQUIVALENT CIRCUIT MODELS FOR COAXIAL OSLT STANDARDS

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ABSTRACT

We use a general description of transmission lines to develop analytic descriptions for offset OSLT (Open-Short-Load-Thru) standards used to calibrate vector network analyzers. We then formulate approximations for coaxial standards that implement the equivalent circuit parameters in common use. A comparison of calibrated verification device measurements demonstrates that our representations agree well with the models used in commercial instruments.

I. INTRODUCTION

In developing software for specialized applications [1], we require full OSLT (Open-Short-Load-Thru) calibration routines such as those used in commercial vector network analyzers (VNA). The literature provides much information on network analyzer error models [2-5] but only limited information on equivalent circuit descriptions of the calibration standards [6,7]. Consequently, we derived a description of offset OSLT calibration standards, starting from a general description of waveguides [8]. We then used this description to approximate coaxial standards based on the same equivalent circuit parameters used in commercial VNAs. We present this model with comparisons of calibrated data, and in doing so, demonstrate a tool for evaluating measurement uncertainty and the limits related to the common equivalent-circuit approach.

To appreciate the need for equivalent circuit models, one must realize that vector network analyzer calibrations, like OSLT, require standards with known characteristics; in particular, the reflection coefficients or scattering parameters. The most direct approach is to calculate VNA correction coefficients using verifiable measurements of the standards. Since it is inconvenient to incorporate direct measurements of the standards into commercial VNA operation, manufacturers typically provide a description of the standards based on equivalent circuit parameters (also known as Calibration Kit Parameters [9] and Calibration Component Coefficients [10]). The majority of VNA users rely on these descriptions, and in doing so, rely on the instrument manufacturer's error analysis to include the limitations of these approximate descriptions.

With coaxial and waveguide standards, the approximations have worked to the satisfaction of most users, even though the available parameters are sometimes used as convenient fitting terms rather than physical descriptors. For on-wafer standards, there we need to understand the physical meaning and potential limitations of the equivalent-circuit parameters. This is particularly true when the offset interconnections are lossy and must be described with complex impedance and complex propagation constants.

The following sections present our description of OSLT calibration standards that include, for the first time, a general treatment of the offset interconnections and Thru standards. The paper extends the description to approximations of coaxial standards. In Sections III and IV, we show this approach to accurately reproduce the corrected data of a commercial vector network analyzer.

II. MODEL

We begin by considering the 12-term error model for a two-port vector network analyzer as shown in Appendix A. Including the isolation terms, there are six forward and six reverse error coefficients to solve for. Standards are connected at the measurement reference planes in place of the device under test (DUT) and, with known values, provide the necessary information to compute each of the error coefficients. Once they are determined, the VNA uses the error coefficients to correct *S*-parameter measurements.

The issue in this paper is not an *n*-term error model for VNAs, but a model of the calibration standards used to compute the expected, or known, *S*-parameters. For our OSLT model we compute expected values of the reflection coefficients for Open, Short, and Load standards, and the *S*-parameters of a two-port Thru standard. We consider these in two parts: a) a generalized description of transmission-line and offset standards; and b) expressions for OSLT standards that incorporate approximations for the transmission lines and terminations and the equivalent-circuit parameters available to commercial VNA users.

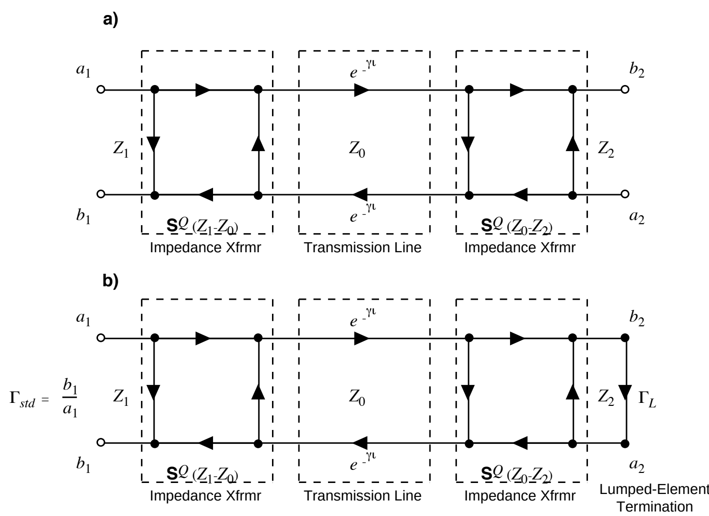

Fig. 1 a) Flow-diagram representation of a uniform transmission line with reference impedance transformers. b) Flow-diagram representation of a lumped-element termination with offset transmission line.

A. General Description

Most VNA calibration kits include a section of transmission line or waveguide, either as one or more of the standards, or in order to distance a termination from the connector region (Fig. 1). Accordingly, we develop our description with a general waveguide approach, starting with a finite-length Thru standard. Marks and Williams [8] show that the cascade matrix $\mathbf{R}^T$ of a single-mode transmission line of characteristic impedance Z_0, connected between Port 1 and Port 2, with reference impedance Z_1 and Z_2, is:

$$\mathbf{R}^T = \mathbf{Q}^{1,0}\mathbf{T}\mathbf{Q}^{0,2}, \tag{1}$$

where $\mathbf{Q}^{1,0}$ and $\mathbf{Q}^{0,2}$ are the cascade matrices transforming complex reference impedances from Z_1 to Z_0, and from Z_0 to Z_2, respectively. In general,

$$\mathbf{Q}^{nm} = \sqrt{\frac{1 - j\,\mathrm{Im}\{Z_m\}/\mathrm{Re}\{Z_m\}}{1 - j\,\mathrm{Im}\{Z_n\}/\mathrm{Re}\{Z_n\}}} \bullet \frac{1}{\sqrt{1-\Gamma_{nm}^2}} \begin{bmatrix} 1 & \Gamma_{nm} \\ \Gamma_{nm} & 1 \end{bmatrix}, \tag{2}$$

where

$$\Gamma_{nm} \equiv \frac{Z_m - Z_n}{Z_m + Z_n}. \tag{3}$$

$\mathbf{T}$ is the cascade matrix of the transmission line with characteristic impedance Z_0:

$$\mathbf{T} = \begin{bmatrix} e^{-\gamma\ell} & 0 \\ 0 & e^{+\gamma\ell} \end{bmatrix}, \tag{4}$$

where γ is the propagation constant of a uniform transmission line of length ℓ. $\mathbf{R}^T$ can be transformed to a scattering matrix, which gives the desired $\mathbf{S}^T$ for the Thru standard:

$$\mathbf{S}^T = \begin{bmatrix} S_{11}^T & S_{12}^T \\ S_{21}^T & S_{22}^T \end{bmatrix} = \frac{1}{R_{22}^T} \begin{bmatrix} R_{12}^T & R_{11}^T R_{22}^T - R_{21}^T R_{12}^T \\ 1 & -R_{21}^T \end{bmatrix}. \tag{5}$$

With known Z_0, Z_1, Z_2, and γ, $\mathbf{S}^T$ provides values necessary to compute the error coefficients of Appendix A. The reference impedance values Z_1 and Z_2 are set by the user, usually taking the same value (50 Ω, for example). For a zero-length Thru in this case, $\mathbf{S}^T$ becomes the identity matrix. In general, however, the length is finite and the transmission line parameters, Z_0 and γ, are complex and frequency dependent.

Equation (5) also provides a description of the offset section of line in the one-port standards. Figure 1 shows a termination (Open, Short, or Load) at the end of an offset length of line. Here, the reflection coefficient Γ_L is known relative to Z_2 (likely the VNA reference impedance), and $\mathbf{S}^T$ describes the offset length with the necessary impedance transformations. The reflection coefficient of the standard, that includes a section of offset line, is:

$$\Gamma_{Std} = S_{11}^T + \frac{S_{21}^T S_{12}^T \Gamma_L}{1 - S_{22}^T \Gamma_L}. \tag{6}$$

These first six equations provide a general description of VNA standards for any complex Z_0, Z_1, Z_2, γ, and Γ_L.

Without loss of generality, Equations (1-5) can be solved analytically to give the scattering parameters of a finite-length Thru when $Z_2 = Z_1$:

$$S_{11} = S_{22} = \frac{\Gamma_Q\left(1 - e^{-2\gamma\ell}\right)}{1 - \Gamma_Q^2 e^{-2\gamma\ell}} \tag{7}$$

$$S_{21} = S_{12} = \frac{\left(1 - \Gamma_Q^2\right) e^{-\gamma\ell}}{1 - \Gamma_Q^2 e^{-2\gamma\ell}}, \tag{8}$$

where

$$\Gamma_Q = \frac{Z_0 - Z_1}{Z_0 + Z_1}. \tag{9}$$

Likewise, we derive an analytic expression for the reflection coefficient of an offset standard Γ_{Std} for the case when the reflection coefficient Γ_L is defined relative to Z_0, that is to say $\mathbf{Q}^{0,2} = \mathbf{Q}^{0,0}$, the identity matrix:

$$\Gamma_{Std} = \frac{\Gamma_Q + \Gamma_L e^{-2\gamma\ell}}{1 + \Gamma_Q \Gamma_L e^{-2\gamma\ell}}. \tag{10}$$

In general,

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}, \tag{11}$$

where Z_L is the impedance of the particular termination.

Equations (1-11) provide a description of VNA standards based on complex impedances and propagation constants, but do not provide determinations of Z_0, γ, and Γ_L. For low-loss coaxial standards, these functions are usually approximated in practice. Without addressing the suitability of such approximations at this point, we develop a model for Z_0, γ, and Γ_L based on the equivalent circuit parameters supplied with commercial OSLT standards.

B. Analytic Solution with Approximations

The common coaxial approximations describe the Open as an effective capacitance C_{eff}, the Short as an effective inductance L_{eff}, and the fixed Load with a single resistance value Z_{Load}. Based on this approach, we approximate Eqn. (11) for each of the three standards. For an Open,

$$\Gamma_{Open} \approx \frac{\dfrac{1}{j\omega C_{eff}} - Z_0}{\dfrac{1}{j\omega C_{eff}} + Z_0} = \frac{1 - j\omega C_{eff} Z_0}{1 + j\omega C_{eff} Z_0}; \tag{12}$$

for a Short,

$$\Gamma_{Short} \approx \frac{j\omega L_{eff} - Z_0}{j\omega L_{eff} + Z_0}; \tag{13}$$

and for a fixed Load,

$$\Gamma_{Load} \approx \frac{Z_{Load} - Z_0}{Z_{Load} + Z_0}, \tag{14}$$

where ω is the angular frequency. Z_{Load} is the frequency-independent load resistance. The effective capacitance of a coaxial Open can be described using a polynomial in frequency (f) [7,11]:

$$C_{eff} = C_0 + C_1 f + C_2 f^2 + C_3 f^3, \tag{15}$$

where C_0-C_3 are fitting coefficients in the units of F, F/Hz, F/Hz^2, and F/Hz^3, respectively. Similarly, the coaxial Short's inductance can be described with a polynomial in units of henry:

$$L_{eff} = L_0 + L_1 f + L_2 f^2 + L_3 f^3. \tag{16}$$

If γ and Z_0 are known, substituting Eqns. (12-14) into Eqn. (10) in place of Γ_L gives the expected reflection coefficients for coaxial, offset Open, Short, and Load standards. If we take Z_0 to be a frequency-independent value, as in commercial VNA calibrations, we can show Eqns. (12-14) to be the same as those identified in Ref. [7]. Although the approximations of Eqns. (12-16) introduce some measurement uncertainty for coaxial standards, Eqn. (10) can accommodate any measured or modeled values for Γ_L, extending the application of OSLT calibrations to other interconnection schemes, such as those encountered in on-wafer measurements.

We next approximate the transmission-line parameters required by Eqns (7-14) using the available model parameters of commercial VNAs. Following the description of transmission-line parameters in Ref. [8], and assuming negligible dielectric loss, the characteristic impedance and propagation constant of a low-loss transmission line may be approximated as:

$$Z_0 = Z_0' + jZ_0'' \approx \sqrt{\frac{L}{C}} - j\frac{R}{2\omega\sqrt{LC}} \tag{17}$$

$$\gamma = \alpha + j\beta \approx \frac{R}{2\sqrt{L/C}} + j\omega\sqrt{LC}\,, \tag{18}$$

where R, L, and C, are the distributed resistance, inductance and capacitance expressed as values per unit length. The available equivalent circuit model parameters in a commercial VNA are t_d, r_{offset}, and Z'_0. Equations (17) and (18) can be cast in terms of the available parameters first by considering r_{offset}, the approximate total conductor resistance divided by the time delay t_d of the transmission line[1]. It is typically specified at 1 GHz and scaled as the square root of frequency over 1 GHz to approximate skin-effect losses. We can approximate R in terms of r_{offset} as:

$$R \approx \frac{r_{offset} t_d}{\ell} = \frac{r_{offset}}{v} = \frac{r_{offset}}{1/\sqrt{LC}}\,, \tag{19}$$

where v is the group velocity on a low-loss transmission line. With this and the user-provided parameter, Z'_0, we obtain:

$$Z_0 = Z_0' - j\frac{r_{offset}}{2\omega}. \tag{20}$$

Likewise the attenuation factor can be cast in terms of r_{offset},

[1] The units of the model parameters C_0-C_3, L_0-L_3, r_{offset}, and t_d are treated generally here but have specific unit multipliers for commercial VNA applications.

$$\alpha \approx \frac{r_{offset} t_d}{2 Z_0' \ell}; \tag{21}$$

and the phase factor in terms of t_d and ℓ,

$$\beta \approx \frac{\omega t_d}{\ell}; \tag{22}$$

to give:

$$\gamma \ell = \alpha \ell + j\beta \ell \approx \frac{r_{offset} t_d}{2 Z_0'} + j\omega t_d. \tag{23}$$

Equations (20 & 23) represent the most significant loss of generality created by the use of the low-loss approximation. It would be inappropriate to use these equations for lossy on-wafer applications, but they can be acceptable for low-loss coaxial standards. Another set of approximations considers the skin-effect resistance and reactance of the conductors, distinguishing the external and internal inductance in the derivation ($R + j\omega L_e + j\omega L_i$). Interestingly, this approach did not agree with the commercial VNA results as well as when we used Eqns. (20 & 23).

III. Experimental Verification

To compare our method to existing algorithms, we performed OSLT calibrations on a commercial vector network analyzer, using 2.4 mm coaxial calibration standards and the VNA model parameters supplied with our set of standards. We then acquired corrected data from four verification artifacts: a 20 dB attenuator, a 40 dB attenuator, a 50 Ω air line, and a 25 Ω mismatch Beatty line. Immediately following these measurements, we collected uncorrected data for the OSLT standards and the verification artifacts to use in testing our algorithms. Next, we computed the expected *S*–parameters of the standards using a software implementation of our model (Eqns. 7-23) and the same equivalent circuit parameters used for the VNA measurements. We then used these computed *S*–parameters and the uncorrected standards measurements to calculate the error coefficients of the 12-term model. We corrected the raw verification device data in our software following Appendix A and compared these data to calibrated measurements made on the VNA. We omitted the isolation terms in both the VNA measurements and our software calibrations.

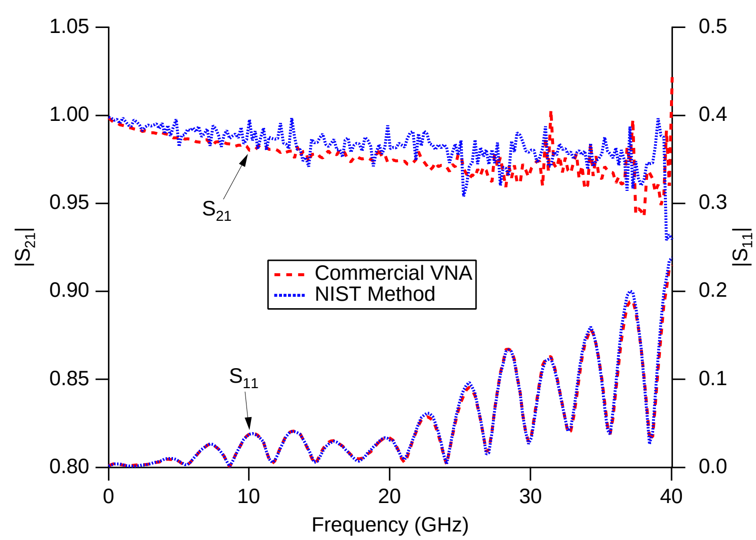

Fig. 2 Comparison of NIST method to commercial VNA calibration for the air line.

Since the two methods could not use identical standards measurements, we consider a bound on the repeatability of our system. The calibration comparison method [12,13] uses the error coefficients from two calibrations to compute a worst-case bound. We performed two sequential OSLT calibrations using the same 2.4 mm standards and instrument configuration as above, and computed the worst-case repeatability bound for comparison [14].

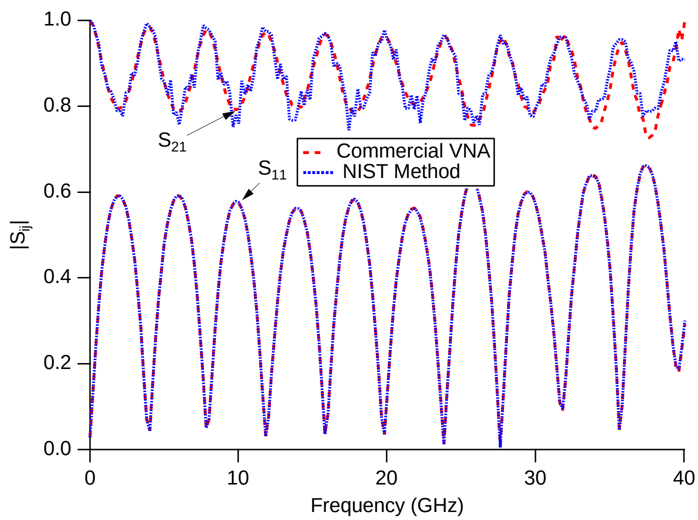

Fig. 3 Comparison of NIST method to commercial VNA calibration for the 25 Ω Beatty line.

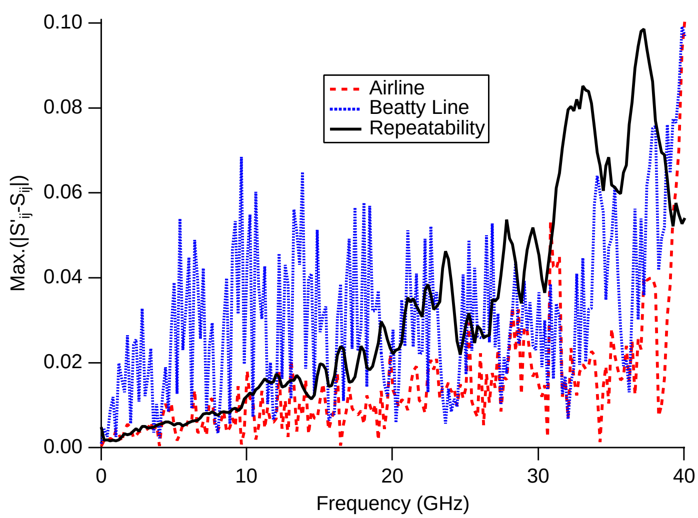

Fig. 4 Comparison of the maximum magnitude of the vector difference between corrected measurements made with our Equivalent-Circuit Model and corrected measurements made with a commercial VNA. Coaxial air line and 25 Ω Beatty line shown compared to instrument-repeatability bound.

The *S*-parameters corrected with our model agree well with the *S*-parameters corrected by the commercial VNA over the 40 GHz bandwidth explored. Figures 2 and 3 compare the $|S_{11}|$ and $|S_{21}|$ data for the air-line and Beatty-line verification artifacts. When comparing magnitude values, the two methods appear to agree very well, particularly in reflection coefficient. To extend the comparison, we look for the maximum in the vector difference $|S'_{ij} - S_{ij}|$, where S'_{ij} is any *S*-parameter corrected with our model, and S_{ij} is that *S*-parameter corrected by the VNA. Figure 4 shows the maximum vector difference for the air line and the Beatty line in comparison to the worst-case repeatability bound. The calibrated air-line data agree, as do the attenuator data not shown here. The maximum differences in the Beatty-line data, however, fall at nearly the same level over the entire frequency span, exceeding the repeatability bound for frequencies less than 20 GHz. Careful observations of the transmission parameter in Fig. 3 shows this discrepancy to be caused by fluctuations in our corrected transmission parameters that fall on either side of the VNA values.

Even though the level of agreement achieved shows our approach to reproduce the calibration of commercial VNAs, the observed noise in our Beatty line data seems to indicate a residual difference between the two. We have examined some of the more obvious sources of this "noise": numerical techniques, calculations of expected *S*-parameters, and raw data from multiple measurements. We have not found the source of this residual difference to date.

We also examined the effect of miscalculated correction factors by implementing an "ideal" calibration in our software. This correction treats the standards as perfect Open, Short, Load and Thru (without offsets) rather than using the equivalent-circuit model developed above. The differences between this ideal calibration and one using the equivalent-circuit parameters is illustrated in Fig. 5 for a coaxial air line. The Ideal Standards curve represents the error due to a gross miscalculation of the equivalent circuit parameters, and the NIST Method curve is the same as that plotted in Fig. 4. Clearly, errors in the approximations have a large influence in the overall measurement uncertainty of the system.

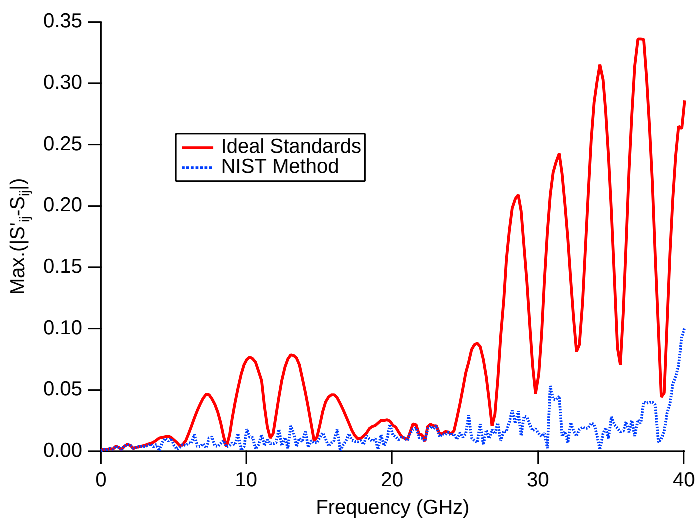

Fig. 5 Illustration of the effects of erroneous model parameters. The "Ideal Standards" curve gives the maximum difference in corrected *S*-parameters of a coaxial air line between treating the OSLT standards as ideal and using an equivalent-circuit model. The "NIST Method" curve shows the differences between the model of this paper and that of a commercial VNA for the air line.

IV. DISCUSSION

Section II A presents a general description of OSLT standards, including standards with lossy transmission line segments. Given accurate measurements or models for the standard's termination and its transmission line parameters, Eqns. (1-11) would give the necessary descriptions to complete the

VNA calibration of Appendix A. The difficulty lies in determining Z_0, γ, and Γ_L for lossy environments, such as in on-wafer measurements.

For low-loss standards, we can make certain simplifying approximations. We verify our method by approximating low-loss coaxial standards in the same manner followed in commercial VNAs [7]. In doing this, we provide a set of equations equivalent to those used in commercial instruments; this was necessary for our own software efforts. With the exception of the "noise" observed in the Beatty-line transmission coefficients, Section III demonstrates the equivalence between our model and commercial VNA operation.

Though not the purpose of the current reference, the model is the basis of exploring limitations in commercial OSLT calibrations applied to on-wafer environments. For example, one can see the loss of generality in adopting Eqns. (12-14). Though it may be possible to describe a coaxial termination with a single effective value (or frequency-polynomial), such representations may be fundamentally flawed in describing on-wafer terminations made from thin metal layers. However, current on-wafer practices make use of the descriptions of Ref. [7] even though these equations were formed following the low-loss approximations outlined in Section II B.

Until we developed our model and its software implementation, we did not have the means to readily study these limitations. Now, we can perform empirical sensitivity studies and possible analytic error analyses. Future work should include a direct comparison of the standards values calculated from any model to measurements of the standards made on a verifiable network analyzer. Such work could directly bound the uncertainty and limits of the commonly used equivalent-circuit models.

V. Summary

We have presented a description for vector network analyzer standards that includes for the first time a generalized approach to offset reflection standards and finite-length Thru standards. This reference provides in one place the equations necessary to perform Open-Short-Load-Thru calibrations for any standards, given either direct measurements or appropriate models for the transmission-line parameters and reflection coefficient of the termination. The paper also extends our description to approximations of low-loss coaxial standards based on the equivalent-circuit parameters available in commercial VNAs. Verification-device data corrected by use of our methods agree well with data corrected by a commercial VNA.

This paper also provides the basis for bounding measurement uncertainty associated with approximate descriptions. Future work should not only examine the discrepancies seen in the Beatty-line data, but bound VNA measurement uncertainty as a function of the approximations employed. This could identify the limits to be expected when using commercial VNA calibrations for lossy standards, such as in on-wafer OSLT calibrations.

Acknowledgments

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APPENDIX A. OSLT CALIBRATION

For thoroughness, we include the equations we used in our VNA calibration. This technique makes use of the twelve-term error model [2,5], shown in Fig. A1. The twelve terms are defined in Table A1, where six terms belong to the forward configuration and six to the reverse configuration.

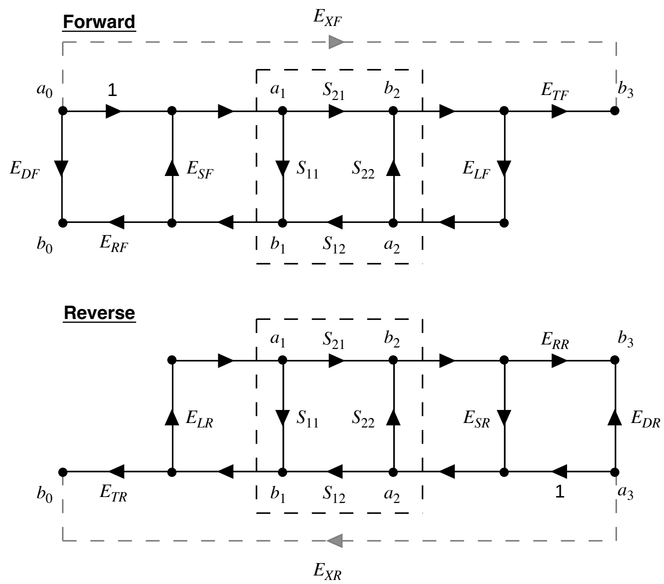

Figure A1. The twelve-term error model with a) forward, and b) reverse configurations.

Table A1. Definitions of the twelve calibration terms.

E_{DF}	Directivity	E_{DR}	Directivity
E_{SF}	Port-1 Match	E_{SR}	Port-2 Match
E_{RF}	Reflection Tracking	E_{RR}	Reflection Tracking
E_{XF}	Leakage	E_{XR}	Leakage
E_{LF}	Port-2 Match	E_{LR}	Port-1 Match
E_{TF}	Transmission Tracking	E_{TR}	Transmission Tracking

First, E_{DF}, E_{SF}, and E_{RF} can be solved by connecting three known terminations Γ_1, Γ_2, and Γ_3, to port 1, and measuring their raw refelction responses (b_0/a_0) Γ_{1M}, Γ_{2M}, and Γ_{3M}:

$$E_{DF} + \Gamma_1\Gamma_{1M}E_{SF} - \Delta E\,\Gamma_1 = \Gamma_{1M} \tag{A1}$$
$$E_{DF} + \Gamma_2\Gamma_{2M}E_{SF} - \Delta E\,\Gamma_2 = \Gamma_{2M} \tag{A2}$$
$$E_{DF} + \Gamma_3\Gamma_{3M}E_{SF} - \Delta E\,\Gamma_3 = \Gamma_{3M}, \tag{A3}$$

where

$$\Delta E = E_{DF}E_{SF} - E_{RF}. \tag{A4}$$

Next, E_{DR}, E_{SR}, and E_{RR} can be solved by connecting the same terminations to port 2:

$$E_{DR} + \Gamma_1\Gamma_{1M}E_{SR} - \Delta E'\,\Gamma_1 = \Gamma_{1M} \tag{A5}$$

$$E_{DR} + \Gamma_2 \Gamma_{2M} E_{SR} - \Delta E' \Gamma_2 = \Gamma_{2M} \tag{A6}$$

$$E_{DR} + \Gamma_3 \Gamma_{3M} E_{SR} - \Delta E' \Gamma_3 = \Gamma_{3M}, \tag{A7}$$

where

$$\Delta E' = E_{DR} E_{SR} - E_{RR}. \tag{A8}$$

Connecting the load terminations to ports 1 and 2 and measuring the raw tranmission parameters S_{21M} (b_3/a_0) and S_{12M} (b_0/a_3) directly gives the isolation terms E_{XF} and E_{XR}, respectively. Then connecting the two ports together with the Thru standard allows the remaining four coefficients to be determined in terms of raw measurements of the Thru S-parameters, S_{11M} (b_0/a_0), S_{21M} (b_3/a_0), S_{12M} (b_0/a_3), and S_{22M} (b_3/a_3):

$$E_{LF} = \frac{S_{11M} - S_{11F}}{S_{22F}\left(S_{11M} - S_{11F}\right) + S_{21F} S_{12F}} \tag{A9}$$

$$E_{TF} = \frac{(S_{21M} - E_{XF})(1 - E_{LF} S_{22F})}{S_{21F}} \tag{A10}$$

$$E_{TR} = \frac{(S_{12M} - E_{XR})(1 - E_{LR} S_{11R})}{S_{12R}} \tag{A11}$$

$$E_{LR} = \frac{S_{22R} - S_{22M}}{S_{11R}\left(S_{22M} - S_{22R}\right) + S_{21R} S_{12R}}, \tag{A12}$$

where

$$S_{11F} = E_{DF} + \frac{E_{RF} S_{11}^T}{1 - E_{SF} S_{11}^T} \tag{A13}$$

$$S_{21F} = \frac{S_{21}^T}{1 - E_{SF} S_{11}^T} \tag{A14}$$

$$S_{12F} = \frac{E_{RF} S_{12}^T}{1 - E_{SF} S_{11}^T} \tag{A15}$$

$$S_{22F} = S_{22}^T + \frac{E_{SF} S_{21}^T S_{12}^T}{1 - E_{SF} S_{11}^T} \tag{A16}$$

$$S_{11R} = S_{11}^T + \frac{E_{SR} S_{21}^T S_{12}^T}{1 - E_{SR} S_{22}^T} \tag{A17}$$

$$S_{21R} = \frac{E_{RR} S_{21}^T}{1 - E_{SR} S_{22}^T} \tag{A18}$$

$$S_{12R} = \frac{S_{12}^T}{1 - E_{SR} S_{22}^T} \tag{A19}$$

$$S_{22R} = E_{DR} + \frac{E_{RR} S_{22}^T}{1 - E_{SR} S_{22}^T}. \tag{A20}$$

Once the twelve terms are calculated, the corrected S-parameters of any passive device may be calculated as follows:

$$S_{11} = \frac{b_1}{a_1} = \frac{\left(\frac{S_{11M} - E_{DF}}{E_{RF}}\right)\left[1 + \left(\frac{S_{22M} - E_{DR}}{E_{RR}}\right) E_{SR}\right] - E_{LF}\left(\frac{S_{21M} - E_{XF}}{E_{TF}}\right)\left(\frac{S_{12M} - E_{XR}}{E_{TR}}\right)}{D} \tag{A21}$$

$$S_{21} = \frac{b_2}{a_1} = \frac{\left(\frac{S_{21M} - E_{XF}}{E_{TF}}\right)\left[1 + \left(\frac{S_{22M} - E_{DR}}{E_{RR}}\right)\left(E_{SR} - E_{LF}\right)\right]}{D} \tag{A22}$$

$$S_{12} = \frac{b_1}{a_2} = \frac{\left(\frac{S_{12M} - E_{XR}}{E_{TR}}\right)\left[1 + \left(\frac{S_{11M} - E_{DF}}{E_{RF}}\right)\left(E_{SF} - E_{LR}\right)\right]}{D} \tag{A23}$$

$$S_{22} = \frac{b_2}{a_2} = \frac{\left(\frac{S_{22M} - E_{DR}}{E_{RR}}\right)\left[1 + \left(\frac{S_{11M} - E_{DF}}{E_{RF}}\right) E_{SF}\right] - E_{LR}\left(\frac{S_{21M} - E_{XF}}{E_{TF}}\right)\left(\frac{S_{12M} - E_{XR}}{E_{TR}}\right)}{D}, \tag{A24}$$

where

$$D = \left[1 + \left(\frac{S_{11M} - E_{DF}}{E_{RF}}\right) E_{SF}\right]\left[1 + \left(\frac{S_{22M} - E_{DR}}{E_{RR}}\right) E_{SR}\right] - \left(\frac{S_{21M} - E_{XF}}{E_{TF}}\right)\left(\frac{S_{12M} - E_{XR}}{E_{TR}}\right) E_{LF} E_{LR}. \tag{A25}$$

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